MODULE 3

Extended Surface Heat Transfer

EXTENDED SURFACES / FINS

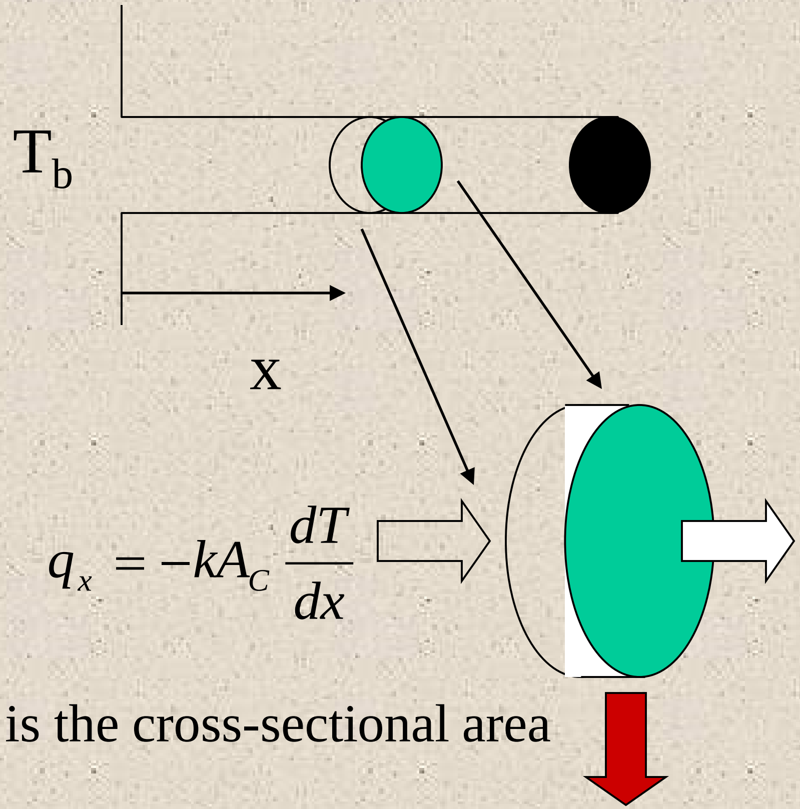
Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $q = hA(T_s-T_\infty)$. Therefore, to increase the convective heat transfer, one can

- ❑ Increase the temperature difference (T_s-T_∞) between the surface and the fluid.

- ❑ Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h. Example: a cooling fan.

- ❑ Increase the contact surface area A. Example: a heat sink with fins.

Extended Surface Analysis



$$q_x = -kA_C \frac{dT}{dx}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

P: the fin perimeter

A_c: the fin cross-sectional area

A_C is the cross-sectional area

$dq_{conv} = h(dA_S)(T - T_\infty)$, where dA_S is the surface area of the element

Energy Balance: $q_x = q_{x+dx} + dq_{conv} = q_x + \frac{dq_x}{dx} dx + hdA_S(T - T_\infty)$

$-kA_C \frac{d^2T}{dx^2} dx + hP(T - T_\infty)dx = 0$, if k, A_C are all constants.

Extended Surface Analysis

(contd….)

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_C}(T - T_\infty) = 0, \text{ A second-order, ordinary differential equation}$$

Define a new variable $\theta(x) = T(x) - T_\infty$, so that

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \text{ where } m^2 = \frac{hP}{kA_C}, \ (D^2 - m^2)\theta = 0$$

Characteristics equation with two real roots: + m & - m

The general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To evaluate the two constants C_1 and C_2, we need to specify two boundary conditions:

The first one is obvious: the base temperature is known as $T(0) = T_b$

The second condition will depend on the end condition of the tip

Extended Surface Analysis (contd...)

For example: assume the tip is insulated and no heat transfer $d\theta/dx(x=L)=0$

The temperature distribution is given by

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh(mL)}$$

The fin heat transfer rate is given by

$$q_f=-kA_C\frac{dT}{dx}(x=0)=\sqrt{hPkA_C}\tanh mL=M\tan mL$$

Temperature distribution for fins of different configurations

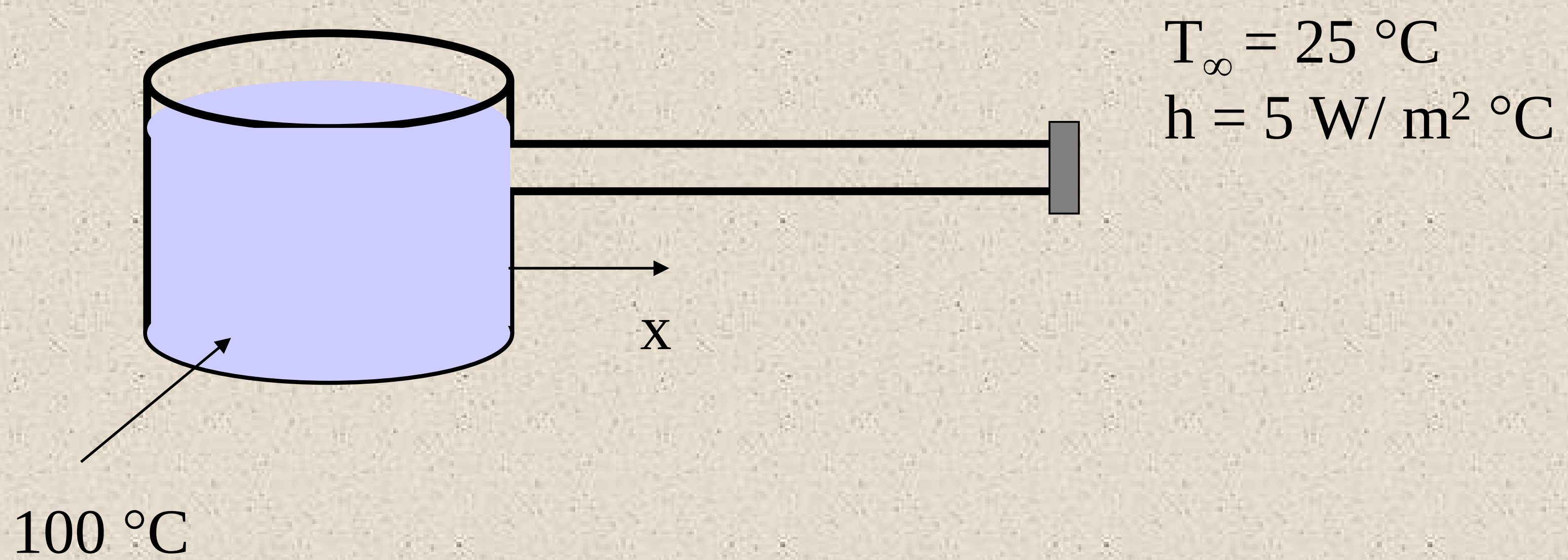
Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\theta_o \dfrac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\theta_0 \tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh m(L-x)+\sinh m(L-x)}{\sinh mL}$	$M\theta_0 \dfrac{(\cosh mL-\theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	$M\ \theta_0$

$$\theta \equiv T-T_\infty, \qquad m^2 \equiv \frac{hP}{kA_C}$$

$$\theta_b=\theta(0)=T_b-T_\infty, \qquad M=\sqrt{hPkA_C}\,\theta_b$$

Example

- An Aluminum pot is used to boil water as shown below. The handle of the pot is 20-cm long, 3-cm wide, and 0.5-cm thick. The pot is exposed to room air at 25°C, and the convection coefficient is 5 W/m^2 °C. Question: can you touch the handle when the water is boiling? (k for aluminum is 237 W/m °C)



Example (contd...)

We can model the pot handle as an extended surface. Assume that there is no heat transfer at the free end of the handle. The condition matches that specified in the fins Table, case B. Use the following data:

$h=5$ W/ m^2 °C, $P=2W+2t=2(0.03+0.005)=0.07$(m), $k=237$ W/m°C, $A_C=Wt=0.00015$(m^2), $L=0.2$(m)

Therefore, $m=(hP/kA_C)^{1/2}=3.138$,

$M=\sqrt{(hPkA_C)}(T_b-T_\infty)=0.111\theta_b=0.111(100-25)=8.325$(W)

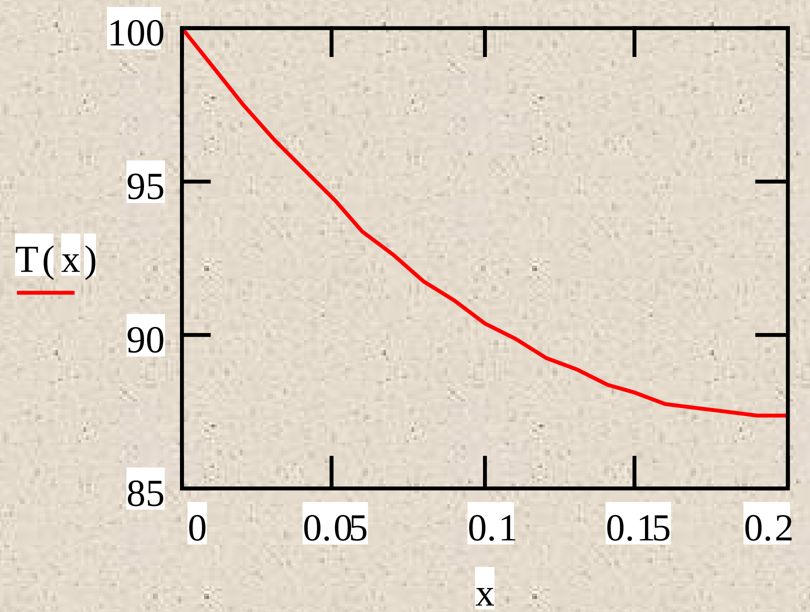
$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$$

$$\frac{T-25}{100-25}=\frac{\cosh[3.138(0.2-x)]}{\cosh(3.138*0.2)},$$

$$T(x)=25+62.32*\cosh[3.138(0.2-x)]$$

Example (contd…)

Plot the temperature distribution along the pot handle



As shown in the figure, temperature drops off but not very steeply. This is because k of aluminium is fairly high.

At the midpoint, T(0.1)=90.4°C. At the end T(0.2)=87.3°C. Therefore, it should not be safe to touch the end of the handle.

The end condition is insulated, hence the gradient is zero.

Example (contd...)

The total heat transfer through the handle can be calculated also. q_f=Mtanh(mL)=8.325tanh[(3.138)(0.2)]=4.632 W

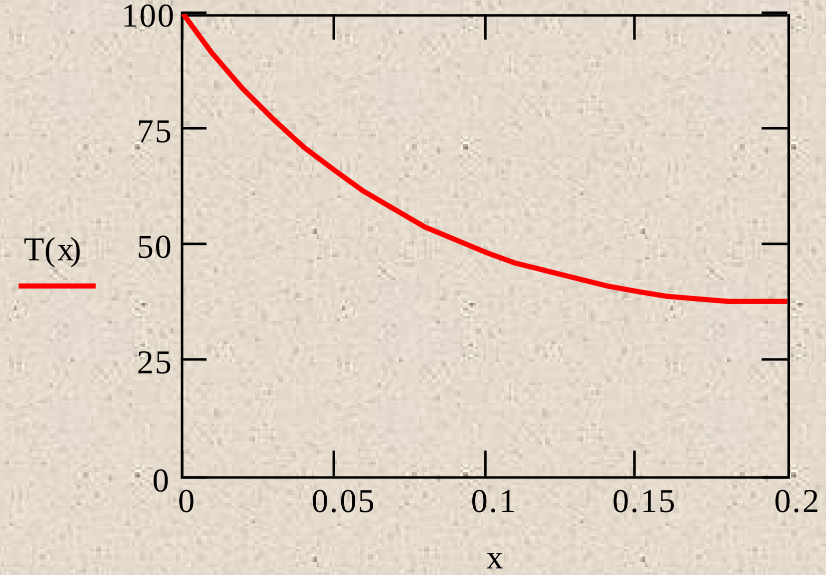
If a stainless steel handle is used instead, what will happen?
For a stainless steel, the thermal conductivity k=15 W/m°C, which is much less compared to aluminium.
Using the same process parameter as before:

$$m = \left(\frac{hP}{kA_C} \right)^{1/2} = 12.47, \quad M = \sqrt{hPkA_C} = 0.0281$$

Example (contd...)

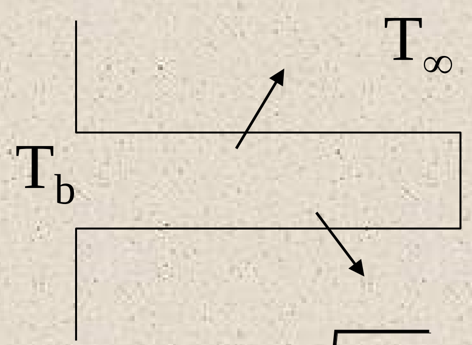
$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)}{\cosh mL}$$

$$T(x)=25+12.3\cosh[12.47(L-x)]$$



Temperature at the handle (x=0.2 m) is only 37.3 °C, not hot at all. This example illustrates the important role of the thermal conductivity of the material in the temperature distribution in a fin.

Fin Design



Total heat loss: q_f=Mtanh(mL) for an adiabatic fin, or q_f=Mtanh(mL$_C$) if there is convective heat transfer at the tip

where $m=\sqrt{\frac{hP}{kA_c}}$, and $M=\sqrt{hPkA_C}\,\theta_b = \sqrt{hPkA_C}\,(T_b - T_\infty)$

Use the thermal resistance concept:

$$q_f = \sqrt{hPkA_C}\tanh(mL)(T_b - T_\infty) = \frac{(T_b - T_\infty)}{R_{t,f}}$$

where $R_{t,f}$ is the thermal resistance of the fin.

For a fin with an adiabatic tip, the fin resistance can be expressed as

$$R_{t,f} = \frac{(T_b - T_\infty)}{q_f} = \frac{1}{\sqrt{hPkA_C}[\tanh(mL)]}$$

Fin Effectiveness

How effective a fin can enhance heat transfer is characterized by the fin effectiveness ε_f: Ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_C(T_b - T_\infty)} = \frac{\sqrt{\mathrm{hPkA_C}}\tanh(mL)}{hA_C} = \sqrt{\frac{kP}{hA_C}}\tanh(mL)$$

If the fin is long enough, mL>2, tanh(mL) $\rightarrow 1$,

it can be considered an infinite fin

$$\varepsilon_f \rightarrow \sqrt{\frac{kP}{hA_C}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

In order to enhance heat transfer, $\varepsilon_f > 1$.

However, $\varepsilon_f \geq 2$ will be considered justifiable

If ε_f <1 then we have an insulator instead of a heat fin

Fin Effectiveness (contd...)

$$\varepsilon_f \to \sqrt{\frac{kP}{hA_C}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

- To increase ε_f, the fin's material should have higher thermal conductivity, k.
- It seems to be counterintuitive that the lower convection coefficient, h, the higher ε_f. But it is not because if h is very high, it is not necessary to enhance heat transfer by adding heat fins. Therefore, heat fins are more effective if h is low. Observation: If fins are to be used on surfaces separating gas and liquid. Fins are usually placed on the gas side. (Why?)

Fin Effectiveness (contd...)

- P/AC should be as high as possible. Use a square fin with a dimension of W by W as an example: P=4W, AC=W2, P/AC=(4/W). The smaller W, the higher the P/AC, and the higher εf.
- Conclusion: It is preferred to use thin and closely spaced (to increase the total number) fins.

Fin Effectiveness (contd...)

The effectiveness of a fin can also be characterized as

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_C(T_b - T_\infty)} = \frac{(T_b - T_\infty)/R_{t,f}}{(T_b - T_\infty)/R_{t,h}} = \frac{R_{t,h}}{R_{t,f}}$$

It is a ratio of the thermal resistance due to convection to the thermal resistance of a fin. In order to enhance heat transfer, the fin's resistance should be lower than that of the resistance due only to convection.

Fin Efficiency

Define Fin efficiency: $\eta_{\mathrm{f}} = \dfrac{q_f}{q_{\max}}$

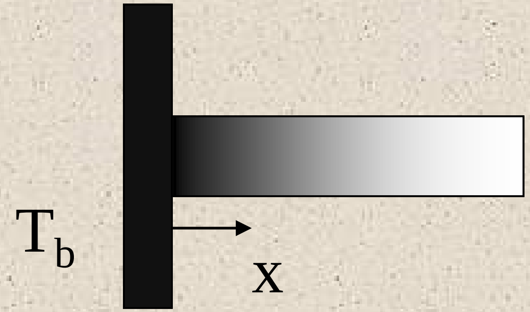
where $q_{\max}$ represents an idealized situation such that the fin is made up of material with infinite thermal conductivity. Therefore, the fin should be at the same temperature as the temperature of the base.

$$q_{\max} = hA_f(T_b - T_\infty)$$

Fin Efficiency (contd…)

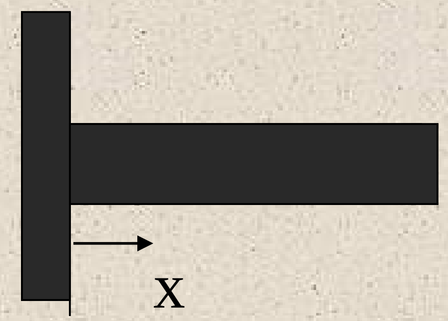
$T(x) < T_b$ for heat transfer to take place

For infinite k $T(x) = T_b$, the heat transfer is maximum



Total fin heat transfer q_f

Real situation



Ideal heat transfer q_{max}

Ideal situation

Fin Efficiency (cont.)

Use an adiabatic rectangular fin as an example:

$$\eta_f = \frac{q_f}{q_{\max}} = \frac{M \tanh mL}{hA_f(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}(T_b - T_\infty)\tanh mL}{hPL(T_b - T_\infty)}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{kA_c}}L} = \frac{\tanh mL}{mL}$$

The fin heat transfer: $q_f = \eta_f q_{\max} = \eta_f hA_f(T_b - T_\infty)$

$$q_f = \frac{T_b - T_\infty}{1/(\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}}, \text{ where } R_{t,f} = \frac{1}{\eta_f hA_f}$$

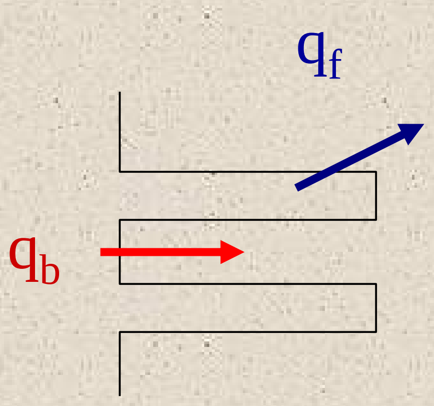
Thermal resistance for a single fin.

As compared to convective heat transfer: $R_{t,b} = \dfrac{1}{hA_b}$

In order to have a lower resistance as that is required to enhance heat transfer: $R_{t,b} > R_{t,f}$ or $A_b < \eta_f A_f$

Overall Fin Efficiency

Overall fin efficiency for an array of fins:



Define terms: A_b: base area exposed to coolant

A_f: surface area of a single fin

A_t: total area including base area and total finned surface, $A_t=A_b+NA_f$

N: total number of fins

Overall Fin Efficiency (contd…)

$$q_t = q_b + Nq_f = hA_b(T_b - T_\infty) + N\eta_f hA_f(T_b - T_\infty)$$

$$= h[(A_t - NA_f) + N\eta_f A_f](T_b - T_\infty) = h[A_t - NA_f(1-\eta_f)](T_b - T_\infty)$$

$$= hA_t[1 - \frac{NA_f}{A_t}(1-\eta_f)](T_b - T_\infty) = \eta_O hA_t(T_b - T_\infty)$$

Define overall fin efficiency: $\eta_O = 1 - \frac{NA_f}{A_t}(1-\eta_f)$

Heat Transfer from a Fin Array

$$q_t = hA_t\eta_O(T_b - T_\infty) = \frac{T_b - T_\infty}{R_{t,O}} \text{ where } R_{t,O} = \frac{1}{hA_t\eta_O}$$

Compare to heat transfer without fins

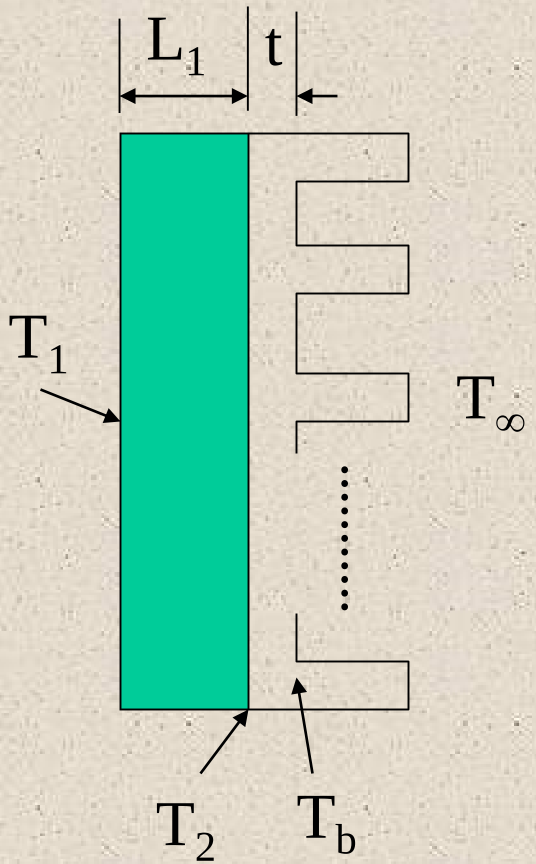
$$q = hA(T_b - T_\infty) = h(A_b + NA_{b,f})(T_b - T_\infty) = \frac{1}{hA}$$

where $A_{b,f}$ is the base area (unexposed) for the fin

To enhance heat transfer $A_t\eta_O >> A$

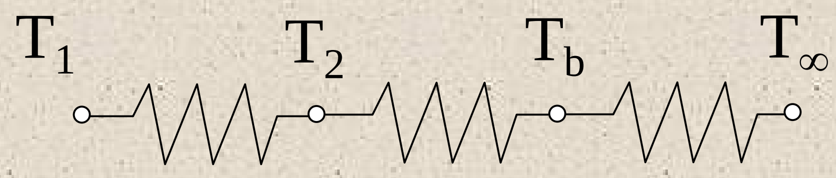
That is, to increase the effective area $\eta_O A_t$.

Thermal Resistance Concept



$$A = A_b + NA_{b,f}$$

$$R_b = t/(k_b A)$$



$$R_1 = L_1/(k_1 A)$$

$$R_{t,O} = 1/(hA_t\eta_O)$$

$$q = \frac{T_1 - T_\infty}{\sum R} = \frac{T_1 - T_\infty}{R_1 + R_b + R_{t,O}}$$